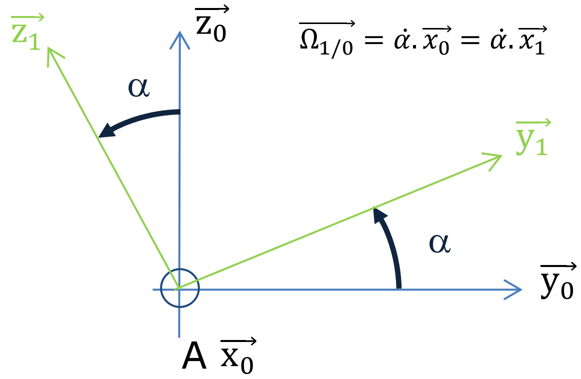
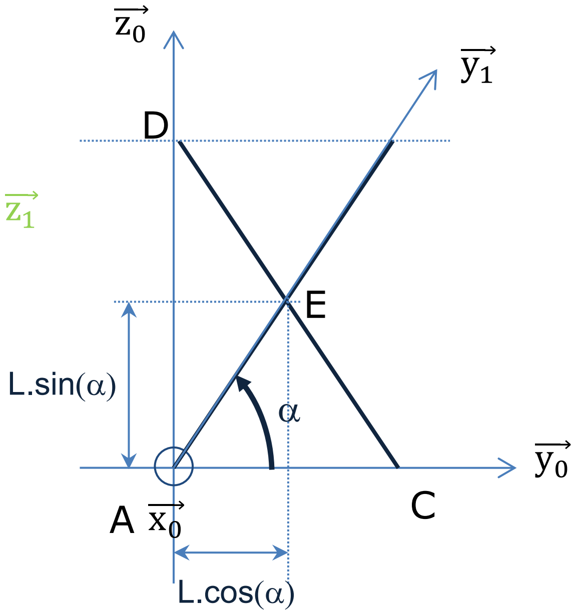
TD : CHAINES OUVERTES – Calcul direct

Exercice 1 : Pont élévateur (éléments de correction)

$$\overrightarrow{\Omega_{1/0}} = \dot{\alpha}.\overrightarrow{x_0} = \dot{\alpha}.\overrightarrow{x_1}$$





$$\overrightarrow{V_{E,1/0}} = \frac{d(\overrightarrow{AE})}{dt} \qquad \overrightarrow{AE} = L.\overrightarrow{y_1}$$

$$\overrightarrow{V_{E,1/0}} = \frac{d}{dt}(L.\overrightarrow{y_1}) = L.\frac{d\overrightarrow{y_1}}{dt}$$

$$\frac{d\overrightarrow{y_1}}{dt} = \overrightarrow{\Omega_{1/0}} \wedge \overrightarrow{y_1} = \dot{\alpha}.\overrightarrow{x_1} \wedge \overrightarrow{y_1} = \dot{\alpha}.\overrightarrow{z_1}$$

$$\boxed{\overrightarrow{V_{E,1/0}} = L.\dot{\alpha}.\overrightarrow{z_1}}$$

$$\overrightarrow{V_{D,2/0}} = \frac{d(\overrightarrow{AD})}{dt} \qquad \overrightarrow{AD} = 2L.\sin(\alpha).\overrightarrow{z_0}$$

$$\overrightarrow{V_{D,2/0}} = \frac{d}{dt}(2L.\sin(\alpha).\overrightarrow{z_0}) = 2L.\dot{\alpha}.\cos(\alpha).\overrightarrow{z_0} + 2L.\sin(\alpha).\frac{d\overrightarrow{z_0}}{dt}$$

$\vec{0}$ car $\overrightarrow{z_0}$ est fixe

$$\boxed{\overrightarrow{V_{D,2/0}} = 2L.\dot{\alpha}.\cos(\alpha).\overrightarrow{z_0}}$$

$$\overrightarrow{V_{C,2/0}} = \frac{d(\overrightarrow{AC})}{dt} \qquad \overrightarrow{AC} = 2L.\cos(\alpha).\overrightarrow{y_0}$$

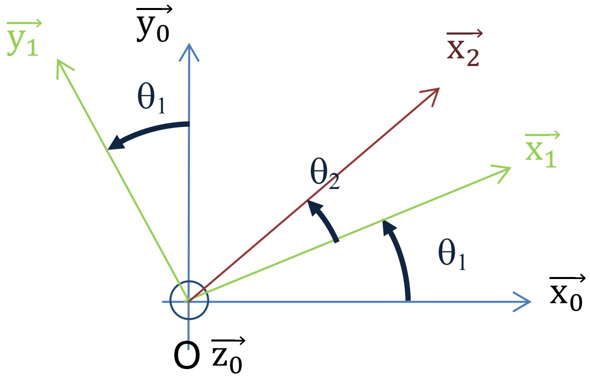
$$\overrightarrow{V_{C,2/0}} = \frac{d}{dt}(2L.\cos(\alpha).\overrightarrow{y_0}) = -2L.\dot{\alpha}.\sin(\alpha).\overrightarrow{y_0} + 2L.\cos(\alpha).\frac{d\overrightarrow{y_0}}{dt}$$

$\vec{0}$ car $\overrightarrow{y_0}$ est fixe

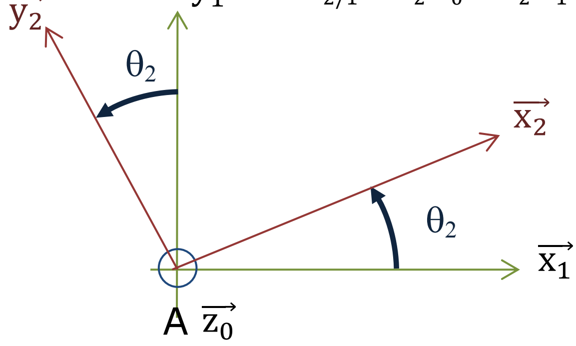
$$\boxed{\overrightarrow{V_{C,2/0}} = -2L.\dot{\alpha}.\sin(\alpha).\overrightarrow{y_0}}$$

Exercice 2 : Préhenseur de pièces (éléments de correction)

$$\overrightarrow{\Omega_{1/0}} = \dot{\theta}_1.\overrightarrow{z_0} = \dot{\theta}_1.\overrightarrow{z_1}$$



$$\overrightarrow{\Omega_{2/1}} = \dot{\theta}_2.\overrightarrow{z_0} = \dot{\theta}_2.\overrightarrow{z_1}$$



$$\overrightarrow{\Omega_{2/0}} = (\dot{\theta}_1 + \dot{\theta}_2).\overrightarrow{z_1}$$

$$\overrightarrow{V_{P,2/0}} = \frac{d(\overrightarrow{OP})}{dt} \qquad \overrightarrow{OP} = a.\overrightarrow{x_1} + b.\overrightarrow{x_2}$$

$$\overrightarrow{V_{P,2/0}} = \frac{d}{dt}(a.\overrightarrow{x_1} + b.\overrightarrow{x_2}) = a.\frac{d\overrightarrow{x_1}}{dt} + b.\frac{d\overrightarrow{x_2}}{dt}$$

$$\frac{d\overrightarrow{x_2}}{dt} = \overrightarrow{\Omega_{2/0}} \wedge \overrightarrow{x_2} = (\dot{\theta}_1 + \dot{\theta}_2).\overrightarrow{z_1} \wedge \overrightarrow{x_2} = (\dot{\theta}_1 + \dot{\theta}_2).\overrightarrow{y_2}$$

$$\frac{d\overrightarrow{x_1}}{dt} = \overrightarrow{\Omega_{1/0}} \wedge \overrightarrow{x_1} = \dot{\theta}_1.\overrightarrow{z_0} \wedge \overrightarrow{x_1} = \dot{\theta}_1.\overrightarrow{y_1}$$

$$\boxed{\overrightarrow{V_{P,2/0}} = a.\dot{\theta}_1.\overrightarrow{y_1} + b.(\dot{\theta}_1 + \dot{\theta}_2).\overrightarrow{y_2}}$$

$$\overrightarrow{a_{P,2/0}} = \frac{d(\overrightarrow{V_{P,2/0}})}{dt}$$

$$\overrightarrow{a_{P,2/0}} = \frac{d}{dt}(a.\dot{\theta}_1.\overrightarrow{y_1} + b.(\dot{\theta}_1 + \dot{\theta}_2).\overrightarrow{y_2}) = a.\ddot{\theta}_1.\overrightarrow{y_1} + a.\dot{\theta}_1.\frac{d\overrightarrow{y_1}}{dt} + b.(\ddot{\theta}_1 + \ddot{\theta}_2).\overrightarrow{y_2} + b.(\dot{\theta}_1 + \dot{\theta}_2).\frac{d\overrightarrow{y_2}}{dt}$$

$$\frac{d\overrightarrow{y_1}}{dt} = \overrightarrow{\Omega_{1/0}} \wedge \overrightarrow{y_1} = \dot{\theta}_1.\overrightarrow{z_0} \wedge \overrightarrow{y_1} = -\dot{\theta}_1.\overrightarrow{x_1}$$

$$\frac{d\overrightarrow{y_2}}{dt} = \overrightarrow{\Omega_{2/0}} \wedge \overrightarrow{y_2} = (\dot{\theta}_1 + \dot{\theta}_2)\overrightarrow{z_1} \wedge \overrightarrow{y_2} = -(\dot{\theta}_1 + \dot{\theta}_2).\overrightarrow{x_2}$$

$$\boxed{\overrightarrow{a_{P,2/0}} = a.\ddot{\theta}_1.\overrightarrow{y_1} - a.\dot{\theta}_1{}^2.\overrightarrow{x_1} + b.(\ddot{\theta}_1 + \ddot{\theta}_2).\overrightarrow{y_2} - b.(\dot{\theta}_1 + \dot{\theta}_2)^2.\overrightarrow{x_2}}$$

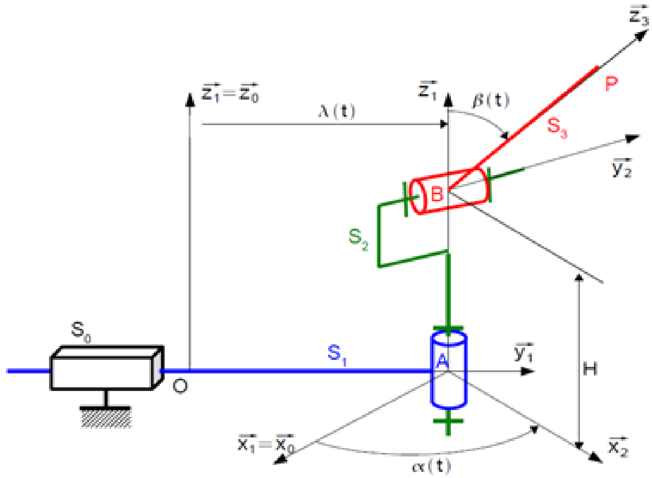
$$\theta_2 = \dot{\theta}_2 = \ddot{\theta}_2 = \ddot{\theta}_1 = 0 \qquad \overrightarrow{a_{P,2/0}} = -a.\dot{\theta}_1{}^2.\overrightarrow{x_1} - b.\dot{\theta}_1{}^2.\overrightarrow{x_2} = -(a+b).\dot{\theta}_1{}^2.\overrightarrow{x_1}$$

car $\overrightarrow{x_1} = \overrightarrow{x_2}$

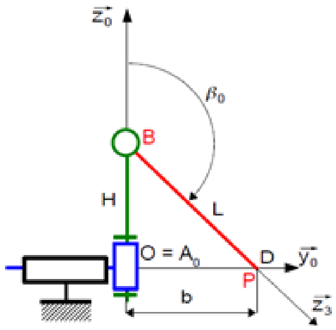
$$\boxed{\left\|\overrightarrow{a_{P,2/0}}\right\| = (a+b).\dot{\theta}_1{}^2 = 51.2\ m/s^2}$$

Le cahier des charges est bien respecté

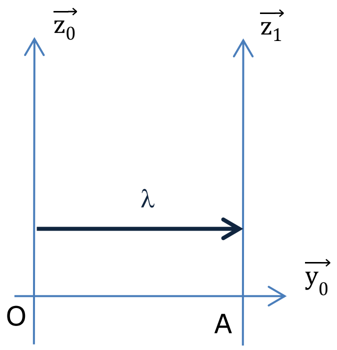
Exercice 4 : Robot de peinture (éléments de correction)



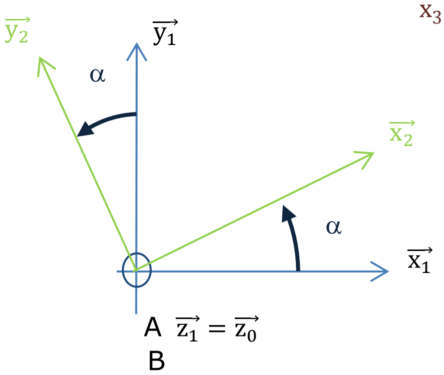
Shéma cinématique du robot



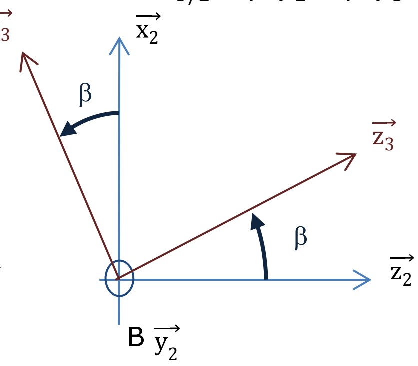
Position médiane (P est en D)



$\overrightarrow{\Omega_{2/1}} = \dot{\alpha}.\overrightarrow{z_1} = \dot{\alpha}.\overrightarrow{z_2}$



$\overrightarrow{\Omega_{3/2}} = \dot{\beta}.\overrightarrow{y_2} = \dot{\beta}.\overrightarrow{y_3}$



La figure en haut à droite rectangle donne dans le triangle OBD : BD.sin(π-β_0) = OP

On déduit : $\sin(\beta_0) = b / L$

$$\overrightarrow{V_{P,3/0}} = \frac{d(\overrightarrow{OP})}{dt} \qquad \overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BP} = \lambda.\overrightarrow{y_0} + H.\overrightarrow{z_1} + L.\overrightarrow{z_3}$$

$$\overrightarrow{V_{P,3/0}} = \frac{d}{dt}(\lambda.\overrightarrow{y_0} + H.\overrightarrow{z_1} + L.\overrightarrow{z_3}) = \dot{\lambda}.\overrightarrow{y_0} + H.\frac{d\overrightarrow{z_1}}{dt} + L.\frac{d\overrightarrow{z_3}}{dt}$$

$\vec{0}$ car $\overrightarrow{z_1} = \overrightarrow{z_0}$ est fixe

$$\frac{d\overrightarrow{z_3}}{dt} = \overrightarrow{\Omega_{3/0}} \wedge \overrightarrow{z_3} = \left(\dot{\alpha}.\overrightarrow{z_2} + \dot{\beta}.\overrightarrow{y_3}\right) \wedge \overrightarrow{z_3} = \dot{\alpha}.\sin(\beta)\,\overrightarrow{y_2} + \dot{\beta}.\overrightarrow{x_3}$$

$$\overrightarrow{V_{P,3/0}} = \dot{\lambda}.\overrightarrow{y_0} + L.\dot{\alpha}.\sin(\beta)\,\overrightarrow{y_2} + L.\dot{\beta}.\overrightarrow{x_3}$$

$\overrightarrow{V_{P,3/0}} = V.\overrightarrow{x_0}$ donne alors les relations (en projetant $\overrightarrow{V_{P,3/0}}$ dans la base b_0) :

$$\begin{cases} -L.\sin(\alpha).\sin(\beta_0) = V \\ \dot{\lambda} = -L.\dot{\alpha}.\cos(\alpha).\sin(\beta_0) \\ \dot{\beta} = 0 \end{cases}$$