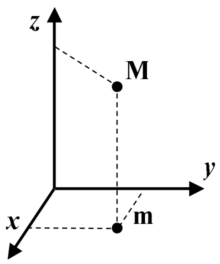
Formulaire d'analyse vectorielle

Calcul vectoriel

$\overrightarrow{\mathrm{rot}}\ \overrightarrow{\mathrm{grad}}\, V = \vec{0}$	$\overrightarrow{\mathrm{grad}}(V_1 V_2) = V_1\, \overrightarrow{\mathrm{grad}}\, V_2 + V_2\, \overrightarrow{\mathrm{grad}}\, V_1$
$\mathrm{div}\, \overrightarrow{\mathrm{rot}}\, \vec{A} = 0$	$\overrightarrow{\mathrm{rot}}(V\vec{A}) = V\, \overrightarrow{\mathrm{rot}}\, \vec{A} + \overrightarrow{\mathrm{grad}}\, V \wedge \vec{A}$
$\mathrm{div}\, \overrightarrow{\mathrm{grad}}\, V = \Delta V$	$\mathrm{div}(V\vec{A}) = V\, \mathrm{div}\, \vec{A} + \overrightarrow{\mathrm{grad}}\, V \cdot \vec{A}$
$\overrightarrow{\mathrm{rot}}\ \overrightarrow{\mathrm{rot}}\, \vec{A} = \overrightarrow{\mathrm{grad}}\, \mathrm{div}\, \vec{A} - \Delta\vec{A}$	$\mathrm{div}(\vec{A}_1 \wedge \vec{A}_2) = \vec{A}_2 \cdot \overrightarrow{\mathrm{rot}}\, \vec{A}_1 - \vec{A}_1 \cdot \overrightarrow{\mathrm{rot}}\, \vec{A}_2$

Coordonnées cartésiennes



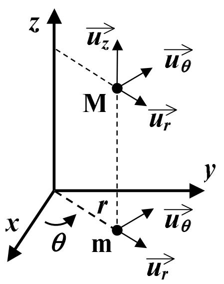
$$\overrightarrow{\mathrm{grad}}\, V = \frac{\partial V}{\partial x}\vec{u}_x + \frac{\partial V}{\partial y}\vec{u}_y + \frac{\partial V}{\partial z}\vec{u}_z$$

$$\mathrm{div}\, \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\overrightarrow{\mathrm{rot}}\, \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\vec{u}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\vec{u}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\vec{u}_z$$

$$\Delta V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Coordonnées cylindriques



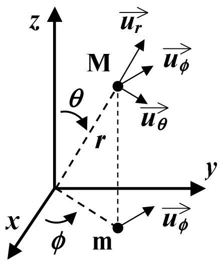
$$\overrightarrow{\mathrm{grad}}\, V = \frac{\partial V}{\partial r}\vec{u}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{u}_\theta + \frac{\partial V}{\partial z}\vec{u}_z$$

$$\mathrm{div}\, \vec{A} = \frac{1}{r}\frac{\partial r A_r}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$$

$$\overrightarrow{\mathrm{rot}}\, \vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right)\vec{u}_r + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\vec{u}_\theta + \frac{1}{r}\left(\frac{\partial r A_\theta}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\vec{u}_z$$

$$\Delta V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{\partial^2 V}{\partial z^2}$$

Coordonnées sphériques



$$\overrightarrow{\mathrm{grad}}\, V = \frac{\partial V}{\partial r}\vec{u}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{u}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\vec{u}_\phi$$

$$\mathrm{div}\, \vec{A} = \frac{1}{r^2}\frac{\partial r^2 A_r}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial \sin\theta A_\theta}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\overrightarrow{\mathrm{rot}}\, \vec{A} = \frac{1}{r\sin\theta}\left(\frac{\partial \sin\theta A_\phi}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right)\vec{u}_r + \ldots$$

$$\ldots + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial r A_\phi}{\partial r}\right)\vec{u}_\theta + \frac{1}{r}\left(\frac{\partial r A_\theta}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\vec{u}_\phi$$

$$\Delta V = \frac{1}{r}\frac{\partial^2 r V}{\partial r^2} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

Théorèmes

Théorème d'Ostrogradsky–Green :
S étant une surface fermée, τ le volume intérieur à S,

$$\oint_{(S)} \vec{A}\cdot \mathrm{d}\vec{S} = \int_{(\tau)} (\mathrm{div}\, \vec{A})\mathrm{d}\tau$$

Théorème de Stokes–Ampère :
C étant une courbe fermée bordant une surface S,

$$\oint_{(C)} \vec{A}\cdot \mathrm{d}\vec{l} = \int_{(S)} (\overrightarrow{\mathrm{rot}}\, \vec{A})\cdot \mathrm{d}\vec{S}$$