Leçon 02 – Correction des ''Exercez-vous''

Exercez vous 3

Déterminer les dérivées partielles secondes des fonctions suivantes :
$f(x, y) = x^3 - y^2x + 2xy + 5$
$f(x, y, z) = xyz^3 - 2x^2 y$
$f(x, y) = \ln(x^2y + y^2)$

Solution

1) $\dfrac{\partial f}{\partial x} = 3x^2 - y^2 + 2y$ $\dfrac{\partial^2 f}{\partial x^2} = 6x$ $\dfrac{\partial^2 f}{\partial x \partial y} = -2y + 2$

$\dfrac{\partial f}{\partial y} = -2yx + 2x$ $\dfrac{\partial^2 f}{\partial y \partial x} = -2y + 2$ $\dfrac{\partial^2 f}{\partial y^2} = -2x$

2) $\dfrac{\partial f}{\partial x} = yz^3 - 4xy$ $\dfrac{\partial^2 f}{\partial x^2} = -4y$ $\dfrac{\partial^2 f}{\partial x \partial y} = z^3 - 4x$ $\dfrac{\partial^2 f}{\partial x \partial z} = 3yz^2$

$\dfrac{\partial f}{\partial y} = xz^3 - 2x^2$ $\dfrac{\partial^2 f}{\partial y \partial x} = z^3 - 4x$ $\dfrac{\partial^2 f}{\partial y^2} = 0$ $\dfrac{\partial^2 f}{\partial y \partial z} = 3xz^2$

$\dfrac{\partial f}{\partial z} = 3xyz^2$ $\dfrac{\partial^2 f}{\partial z \partial x} = 3yz^2$ $\dfrac{\partial^2 f}{\partial z \partial y} = 3xz^2$ $\dfrac{\partial^2 f}{\partial z^2} = 6xyz$

3) $\dfrac{\partial f}{\partial x} = \dfrac{2xy}{x^2y + y^2}$ $\dfrac{\partial^2 f}{\partial x^2} = \dfrac{2y(x^2y + y^2) - 2xy(2xy)}{(x^2y + y^2)^2} = \dfrac{-2x^2y^2 + 2y^3}{(x^2y + y^2)^2}$

$\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{2x(x^2y + y^2) - (x^2 + 2y)(2xy)}{(x^2y + y^2)^2} = \dfrac{-2xy^2}{(x^2y + y^2)^2}$

$\dfrac{\partial f}{\partial y} = \dfrac{x^2 + 2y}{x^2y + y^2}$ $\dfrac{\partial^2 f}{\partial y \partial x} = \dfrac{2x(x^2y + y^2) - 2xy(x^2 + 2y)}{(x^2y + y^2)^2} = \dfrac{-2xy^2}{(x^2y + y^2)^2}$

$\dfrac{\partial^2 f}{\partial y^2} = \dfrac{2(x^2y + y^2) - (x^2 + 2y)^2}{(x^2y + y^2)^2} = \dfrac{-2x^2y - x^4 - 2y^2}{(x^2y + y^2)^2}$