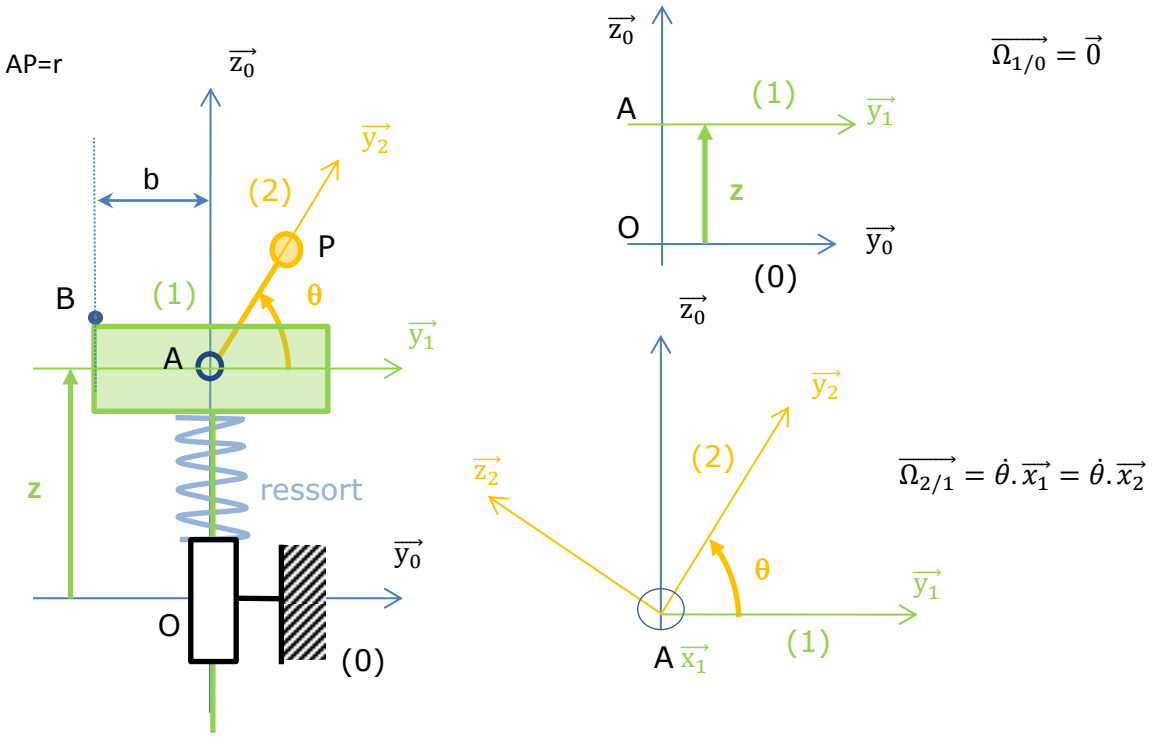
TD : CHAINES OUVERTES – Différentes méthodes

Exercice 1 : Table vibrante (éléments de correction)



$$\overrightarrow{V_{P,2/0}} = \overrightarrow{V_{P,2/1}} + \overrightarrow{V_{P,1/0}}$$

2/1
Rotation autour de A$\overrightarrow{x_1}$

$$\overrightarrow{V_{P,2/1}} = \overrightarrow{\Omega_{2/1}} \wedge \overrightarrow{AP}$$
$$= \dot{\theta}.\overrightarrow{x_2} \wedge r.\overrightarrow{y_2} = r.\dot{\theta}.\overrightarrow{z_2}$$

1/0
Translation suivant $\overrightarrow{z_0}$

$$\overrightarrow{V_{P,1/0}} = \dot{z}.\overrightarrow{z_0}$$

$$\boxed{\overrightarrow{V_{P,2/0}} = \dot{z}.\overrightarrow{z_0} + r.\dot{\theta}.\overrightarrow{z_2}}$$

$$\overrightarrow{a_{P,2/0}} = \frac{d(\overrightarrow{V_{P,2/0}})}{dt}$$

$$\overrightarrow{a_{P,2/0}} = \frac{d}{dt}\left(\dot{z}.\overrightarrow{z_0} + r.\dot{\theta}.\overrightarrow{z_2}\right) = \ddot{z}.\overrightarrow{z_0} + r.\ddot{\theta}.\overrightarrow{z_2} + r.\dot{\theta}.\frac{d\overrightarrow{z_2}}{dt}$$

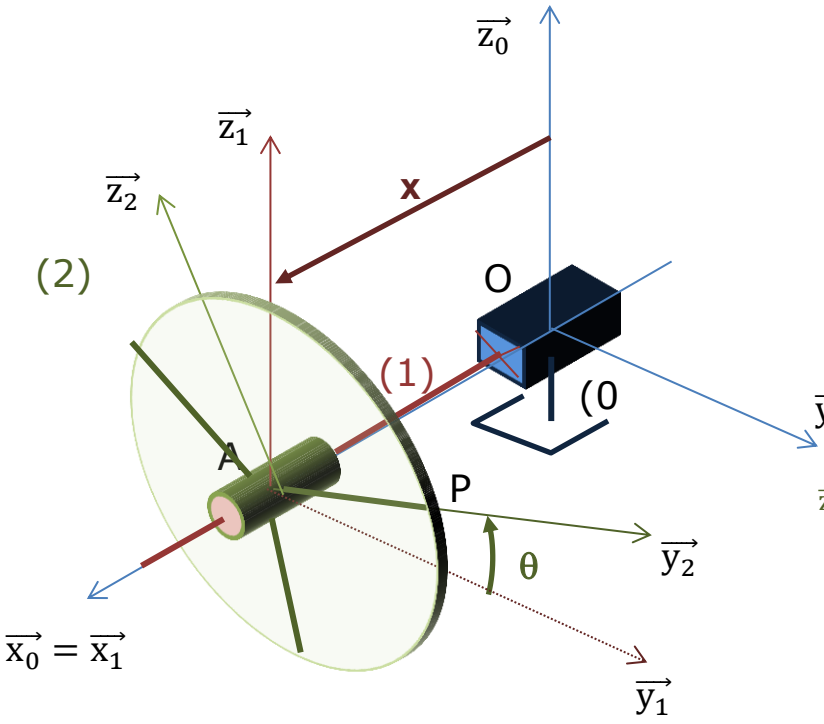
$$\frac{d\overrightarrow{z_2}}{dt} = \overrightarrow{\Omega_{2/0}} \wedge \overrightarrow{z_2} = \dot{\theta}.\overrightarrow{x_2} \wedge \overrightarrow{z_2} = -\dot{\theta}.\overrightarrow{y_2}$$

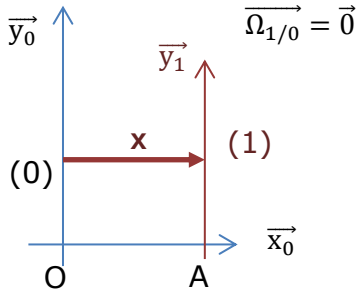
$$\boxed{\overrightarrow{a_{P,2/0}} = \ddot{z}.\overrightarrow{z_0} + r.\ddot{\theta}.\overrightarrow{z_2} - r.\dot{\theta}^2.\overrightarrow{y_2}}$$

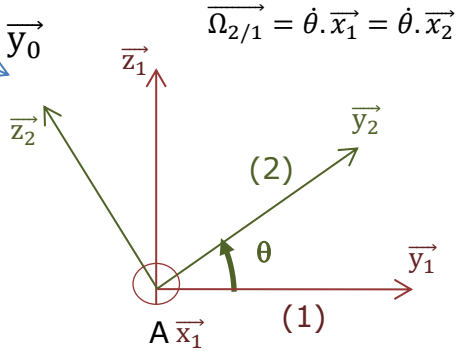
accélération de 1/0

accélération tangentielle et centripète de 2/1

Exercice 2 : Véhicule éolien (éléments de correction)







$$\overrightarrow{V_{P,2/0}} = \overrightarrow{V_{P,2/1}} + \overrightarrow{V_{P,1/0}}$$

2/1
Rotation autour de A$\overrightarrow{x_1}$

$$\overrightarrow{V_{P,2/1}} = \overrightarrow{\Omega_{2/1}} \wedge \overrightarrow{AP}$$
$$= \dot{\theta}.\overrightarrow{x_2} \wedge r.\overrightarrow{y_2} = r.\dot{\theta}.\overrightarrow{z_2}$$

1/0
Translation suivant $\overrightarrow{x_0}$

$$\overrightarrow{V_{P,1/0}} = \dot{x}.\overrightarrow{x_0}$$

$$\boxed{\overrightarrow{V_{P,2/0}} = \dot{x}.\overrightarrow{x_0} + r.\dot{\theta}.\overrightarrow{z_2}}$$

$$\overrightarrow{a_{P,2/0}} = \frac{d(\overrightarrow{V_{P,2/0}})}{dt}$$

$$\overrightarrow{a_{P,2/0}} = \frac{d}{dt}(\dot{x}.\overrightarrow{x_0} + r.\dot{\theta}.\overrightarrow{z_2}) = \ddot{x}.\overrightarrow{x_0} + r.\ddot{\theta}.\overrightarrow{z_2} + r.\dot{\theta}.\frac{d\overrightarrow{z_2}}{dt}$$

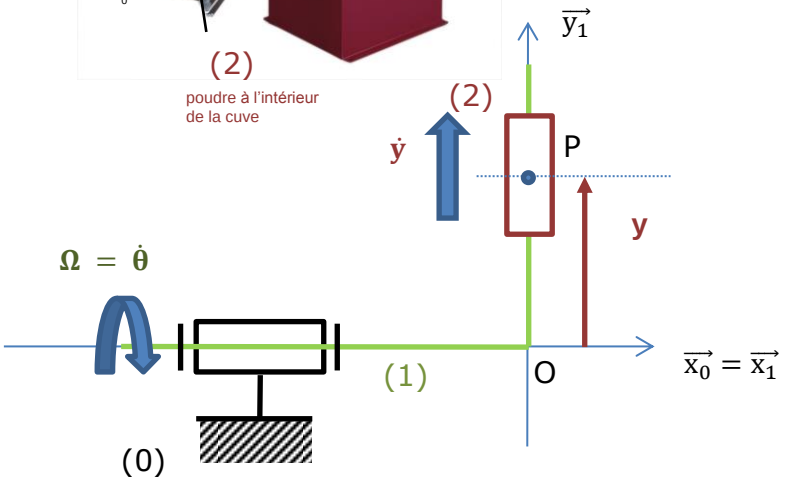
$$\frac{d\overrightarrow{z_2}}{dt} = \overrightarrow{\Omega_{2/0}} \wedge \overrightarrow{z_2} = \dot{\theta}.\overrightarrow{x_2} \wedge \overrightarrow{z_2} = -\dot{\theta}.\overrightarrow{y_2}$$

$$\boxed{\overrightarrow{a_{P,2/0}} = \ddot{x}.\overrightarrow{x_0} + r.\ddot{\theta}.\overrightarrow{z_2} - r.\dot{\theta}^2.\overrightarrow{y_2}}$$

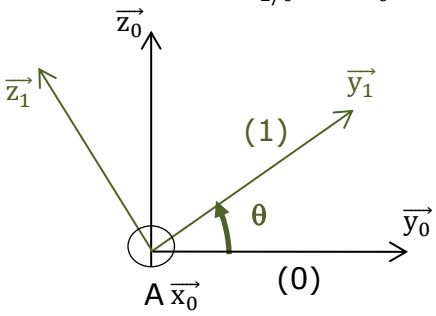
accélération de 1/0

accélération tangentielle et centripète de 2/1

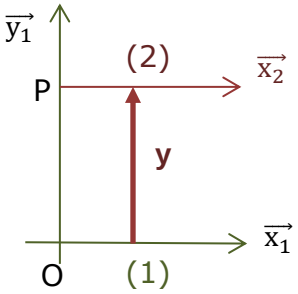
Exercice 3 : Mélangeur de produits (éléments de correction)



$\overrightarrow{\Omega_{1/0}} = \dot{\theta}.\vec{x_0} = \dot{\theta}.\vec{x_1}$



$\overrightarrow{\Omega_{2/1}} = \vec{0}$



$$\overrightarrow{V_{P,2/0}} = \overrightarrow{V_{P,2/1}} + \overrightarrow{V_{P,1/0}}$$

2/1

Translation suivant $\vec{y_1}$

$\overrightarrow{V_{P,2/1}} = \dot{y}.\vec{y_1}$

1/0

Rotation autour de $O\vec{x_0}$

$\overrightarrow{V_{P,1/0}} = \overrightarrow{\Omega_{1/0}} \wedge \overrightarrow{OP}$

$= \dot{\theta}.\vec{x_1} \wedge y.\vec{y_1} = y.\dot{\theta}.\vec{z_1}$

$$\boxed{\overrightarrow{V_{P,2/0}} = y.\dot{\theta}.\vec{z_1} + \dot{y}.\vec{y_1}}$$

$$\overrightarrow{a_{P,2/0}} = \frac{d(\overrightarrow{V_{P,2/0}})}{dt}$$

$$\overrightarrow{a_{P,2/0}} = \frac{d}{dt}(y.\dot{\theta}.\vec{z_1} + \dot{y}.\vec{y_1}) = \left[y.\ddot{\theta}.\vec{z_1} + y.\dot{\theta}.\frac{d\vec{z_1}}{dt} + \dot{y}.\dot{\theta}.\vec{z_1}\right] + \left[\dot{y}.\frac{d\vec{y_1}}{dt} + \ddot{y}.\vec{y_1}\right]$$

$$\frac{d\vec{z_1}}{dt} = \overrightarrow{\Omega_{1/0}} \wedge \vec{z_1} = \dot{\theta}.\vec{x_1} \wedge \vec{z_1} = -\dot{\theta}.\vec{y_1}$$

$$\frac{d\vec{y_1}}{dt} = \overrightarrow{\Omega_{1/0}} \wedge \vec{y_1} = \dot{\theta}.\vec{z_1}$$

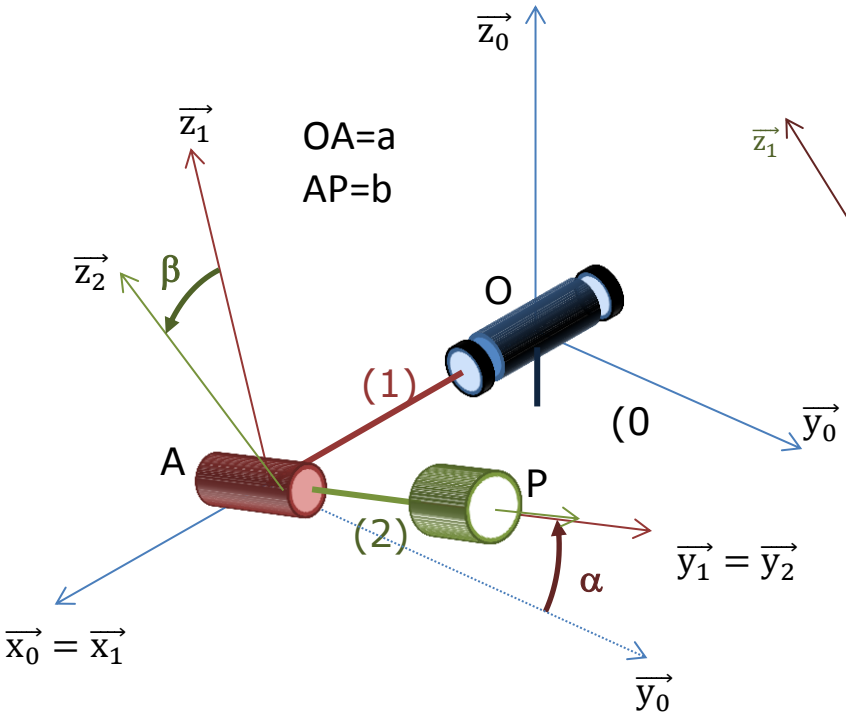
$$\boxed{\overrightarrow{a_{P,2/0}} = y.\ddot{\theta}.\vec{z_1} - y.\dot{\theta}^2.\vec{y_1} + \ddot{y}.\vec{y_1} + 2.\dot{y}.\dot{\theta}.\vec{z_1}}$$

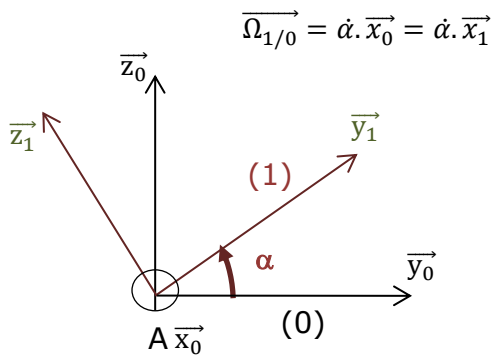
accélération tangentielle et centripète de 2/1

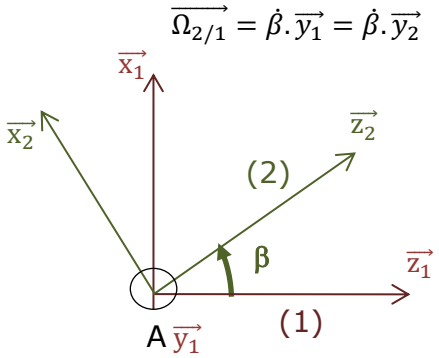
accélération de coriolis

accélération de 1/0

Exercice 4 : soufflante orientable (éléments de correction)







$$\overrightarrow{V_{P,2/0}} = \overrightarrow{V_{P,2/1}} + \overrightarrow{V_{P,1/0}}$$

2/1
Rotation autour de A $\overrightarrow{y_1}$

$$\overrightarrow{V_{P,1/0}} = \overrightarrow{\Omega_{1/0}} \wedge \overrightarrow{OP}$$

$$= \dot{\beta}.\overrightarrow{y_1} \wedge b.\overrightarrow{y_1} = \vec{0}$$

1/0
Rotation autour de A $\overrightarrow{x_0}$

$$\overrightarrow{V_{P,1/0}} = \overrightarrow{\Omega_{1/0}} \wedge \overrightarrow{AP}$$

$$= \dot{\alpha}.\overrightarrow{x_1} \wedge b.\overrightarrow{y_1} = b.\dot{\alpha}.\overrightarrow{z_1}$$

$$\boxed{\overrightarrow{V_{P,2/0}} = b.\dot{\alpha}.\overrightarrow{z_1}}$$

$$\overrightarrow{a_{P,2/0}} = \frac{d(\overrightarrow{V_{P,2/0}})}{dt}$$

$$\overrightarrow{a_{P,2/0}} = \frac{d}{dt}(b.\dot{\alpha}.\overrightarrow{z_1}) = \left[b.\ddot{\alpha}.\overrightarrow{z_1} + b.\dot{\alpha}.\frac{d\overrightarrow{z_1}}{dt}\right]$$

$$\frac{d\overrightarrow{z_1}}{dt} = \overrightarrow{\Omega_{1/0}} \wedge \overrightarrow{z_1} = \dot{\alpha}.\overrightarrow{x_1} \wedge \overrightarrow{z_1} = -\dot{\alpha}.\overrightarrow{y_1}$$

$$\boxed{\overrightarrow{a_{P,2/0}} = b.\ddot{\alpha}.\overrightarrow{z_1} + b.\dot{\alpha}^2.\overrightarrow{y_1}}$$

accélération tangentielle et centripète de 1/0

accélération de 2/1 nulle
(P est sur l'axe de rotation de 2/1)